JECA-2021

MATHEMATICS 1061003049

(Booklet Number)

Duration: **2 Hours** Full Marks: **100**

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full mark **1**. In case of incorrect answer or any combination of more than one answer, ¼ marks will be deducted.
2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C, or D.
3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.
4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.
5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.
6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.
7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/ signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.
8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.
9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.
10. Hand over the OMR to the invigilator before leaving the Examination Hall.

JECA-2021

SPACE FOR ROUGH WORK

MATHEMATICS

1. If α, β, γ are the roots of the equation $x^3 + 5x - 1 = 0$, then the equation whose roots are $\frac{1}{\beta+\gamma}, \frac{1}{\gamma+\alpha}, \frac{1}{\alpha+\beta}$ is

(A) $x^3 + 5x^2 + 1 = 0$
(B) $x^3 - 5x^2 + 1 = 0$
(C) $x^3 - 5x^2 - 1 = 0$
(D) $x^3 + 5x^2 - 1 = 0$

2. The term independent of x and y in the expansion of

$$\begin{vmatrix} x^2+y^2+1 & x^2+2y^2+3 & x^2+3y^2+4 \\ y^2+2 & 2y^2+6 & 3y^2+8 \\ y^2+1 & 2y^2+3 & 3y^2+4 \end{vmatrix}$$ is

(A) 8
(B) 16
(C) 12
(D) 0

3. The rank of the matrix $\begin{bmatrix} 1 & 4 & 2 & 1 \\ 2 & 7 & 5 & 2k \\ 4 & m & 0 & 2k+1 \end{bmatrix}$ is 2. Then (m, k) =

(A) $\left(24, \frac{19}{18}\right)$
(B) $\left(24, -\frac{19}{18}\right)$
(C) $\left(\frac{19}{18}, 24\right)$
(D) $\left(-24, -\frac{19}{18}\right)$

4. If $\log_e(1 + i) = \frac{1}{2}\log_e 2 + k\frac{\pi}{4}i$, then k =

(A) 2n
(B) 2n + 1
(C) 4n + 1
(D) 8n + 1

5. The number of real root(s) of the equation $x^{11} + x^9 + x^7 + x^5 + x^3 + x - 71 = 0$ is

(A) 1
(B) 2
(C) 3
(D) 0

6. If P is a 3 × 3 matrix over ℝ such that $P^T = 2P + I$ when P^T is the transpose of P and I is a 3 × 3 identity matrix, then there exists a non-zero column matrix X such that PX is equal to

(A) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) X

(C) –X (D) 2X

7. Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \binom{m}{1} & \binom{m+1}{1} & \binom{m+2}{1} \\ \binom{m+1}{2} & \binom{m+2}{2} & \binom{m+3}{2} \end{vmatrix}$

Then, Δ =

(A) 1 (B) m

(C) m^2 (D) m^3

8. Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad \neq 0 \right\}$ be a group under matrix multiplication (*).

Let $N = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$, then (N,*) is

(A) a subgroup of (G, *) (B) not a subgroup (G, *)

(C) only a semigroup (D) not even a groupoid

9. If A is a skew-symmetric matrix of order n, B is a column matrix of order n × 1, then $B^T AB$ is a

(A) symmetric matrix (B) skew-symmetric matrix

(C) unit matrix (D) null matrix

10. If $z = \left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ then

(A) $Re(z) = 0$
(B) $Re(z) < 0$ and $Im(z) = 0$
(C) $Re(z) > 0$ and $Im(z) > 0$
(D) $Re(z) > 0$ and $Im(z) < 0$

11. If α is a complex number satisfying the equation $\alpha^2 + \alpha + 1 = 0$ then α^{31} is equal to

(A) α
(B) α^2
(C) 1
(D) i

12. Let (G,.) be a group & $c \in G$. The binary operation $\circ$ be defined on G by $a \circ b = a.c.b^{-1}$ $\forall$ $a, b \in G$, then

(A) $(G, \circ)$ is not a group
(B) $(G, \circ)$ is a group
(C) $(G, \circ)$ is only a semigroup
(D) $(G, \circ)$ is not a semigroup

13. On $\mathbb{R} \times \mathbb{R}$, the operation $*$ be defined as follows :

$(a, b) * (c, d) = (a + c, b + d + 2bd)$ $\forall$ $a, b, c, d \in \mathbb{R}$. Then $(\mathbb{R} \times \mathbb{R}, *)$ is

(A) a group
(B) not a group
(C) semi group but no identity
(D) not a semigroup

14. Let $S = \{-1, 1\}$. On $S \times S$, the operation $*$ be defined as co-ordinate multiplication. Then $(S \times S, *)$ is

(A) a cyclic group
(B) a non-abelian group
(C) a non-cyclic, abelian group
(D) a semigroup only

15. Let $(G, *)$ be an abelian group. Let operation $\cdot$ be defined on G by $a \cdot b = b$ $\forall a, b \in G$. Then $(G, *, \cdot)$ is

(A) Ring
(B) Field
(C) Integral domain
(D) Not a ring

16. Let $\alpha_1 = (2, -3, 1)$, $\alpha_2 = (3, 0, 1)$, $\alpha_3 = (0, 2, 1)$, $\alpha_4 = (1, 1, 1)$ & S be spanned by $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. Then

(A) $\{\alpha_1, \alpha_2, \alpha_4\}$ is a basis of S.

(B) $\{\alpha_1, \alpha_3, \alpha_4\}$ is a basis of S.

(C) $\{\alpha_1, \alpha_2, \alpha_3\}$ is a basis of S.

(D) $\{\alpha_2, \alpha_3, \alpha_4\}$ is a basis of S.

17. Given $\begin{pmatrix} x+y & y-z \\ s-t & 7+x \end{pmatrix} = \begin{pmatrix} t-x & z-t \\ z-y & x+z+t \end{pmatrix}$ then (x, y, z, t) is

(A) (4, 3, 2, 1) (B) (4, 2, 3, 1)

(C) (1, 2, 3, 4) (D) (2, 3, 4, 1)

18. Locus of z for which $|z-8| + |z+8| = 20$ represents a conic section whose eccentricity is

(A) $\frac{1}{5}$ (B) $\frac{5}{2}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

19. If α, β are the roots of the equation $x^2 - 2x \cos\theta + 1 = 0$, then the equation whose roots are α^n, β^n is

(A) $x^2 - 2x \cos n\theta + 1 = 0$ (B) $x^2 + 2x \cos n\theta + 1 = 0$

(C) $x^2 - 2x \sin n\theta + 1 = 0$ (D) $x^2 + 2x \sin n\theta + 1 = 0$

20. If $\alpha_0, \alpha_1, \alpha_2 \ldots \alpha_6$ be the roots of the equation $x^7 - 1 = 0$, then $\sum_{i=0}^{6} \alpha_i^{99} =$

(A) 1 (B) 0

(C) −1 (D) 7

21. The equation $\tan\left\{i\log\frac{x-iy}{x+iy}\right\}=2$ represents

(A) a parabola
(B) an ellipse
(C) a hyperbola
(D) a circle

22. The general values of i^i lie on the

(A) real axis
(B) imaginary axis
(C) line $x = y$
(D) line $x = -y$

23. The equation $3x^5 - 4x^2 + 8 = 0$ has

(A) exactly two imaginary roots
(B) atleast two imaginary roots
(C) exactly two real roots
(D) no negative real roots

24. If the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ are in G.P., then

(A) $b^2 = ad$
(B) $a^2 = c^2d$
(C) $c^2 = a^2d$
(D) $d^2 = ac$

25. Consider the circles $x^2 + y^2 + 5x - 3y + 1 = 0$ and $2x^2 + 2y^2 + 6x - 4y + 1 = 0$

(A) They have no common chord
(B) They have a pair of common chords
(C) They have unique common chord
(D) They touch each other externally

26. Let a and b in the equation $ax + by + c = 0$ vary subject to the condition that $a + b$ is constant. Then

(A) the lines are perpendicular to each other
(B) the lines pass through a fixed point
(C) the lines are parallel to each other
(D) the lines have no special property

27. Transforming to parallel axes through the point (2, –3), the equation $2x^2 + 3xy + 4y^2 + x + 18y + 25 = 0$ is changed to

(A) $2x^2 + 3xy + 4y^2 = 1$
(B) $3x^2 - 11xy + 5y^2 = 2$
(C) $x^2 + 2y^2 = 3$
(D) $2x^2 - y^2 = 11$

28. Tangents are drawn from any point on $x + 4a = 0$ to the parabola $y^2 = 4ax$, then the angle subtend by their chord of contact at the vertex is

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

29. If one of the lines $ax^2 + 2hxy + by^2 = 0$ makes the same angle with the x-axis as the other makes with y-axis, then

(A) $a = b$
(B) $a + b = 0$
(C) $a + b = 1$
(D) $a = b + 1$

30. If O be the center of a circle and P is any point, then OP and the polar of P with respect to the circle are inclined at an angle

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{6}$

31. The equation $4xy - 3x^2 = 1$ is transformed to $x^2 - 4y^2 = 1$ by rotating the axis through an angle θ, where tan θ is equal to

(A) $\frac{1}{2}$
(B) 2
(C) $\frac{1}{3}$
(D) 3

32. The equation of the straight lines through the origin, each of which makes an angle α with the straight line $y = x$ is

(A) $x^2 + 2xy \cos 2\alpha + y^2 = 0$
(B) $x^2 + 2xy \sec 2\alpha + y^2 = 0$
(C) $x^2 - 2xy \sec 2\alpha + y^2 = 0$
(D) $x^2 - 2xy \cos 2\alpha + y^2 = 0$

33. The value of a, for which $ax^2 - 20xy + 25y^2 - 14x + 4y - 15 = 0$ represents a non-central conic, is

(A) 2
(B) 3
(C) 1
(D) 4

34. Locus of the poles of the tangents to the parabola $y^2 = 4bx$ with respect to the parabola $y^2 = 4ax$ is

(A) a straight line
(B) a parabola
(C) an ellipse
(D) a hyperbola

35. The line $\frac{1}{r} = a \cos \theta + b \sin \theta$ touches the circle $r = 2k \cos \theta$ if

(A) $b^2k^2 - 2ak = 1$
(B) $a^2k^2 - 2bk = 1$
(C) $b^2k^2 + 2ak = 1$
(D) $a^2k^2 + 2bk = 1$

36. If $|\vec{\alpha}| = 2$, $|\vec{\beta}| = 3$ and $\vec{\alpha}$, $\vec{\beta}$ are mutually perpendicular, then the area of the triangle having sides $\vec{\alpha} + \vec{\beta}$ and $\vec{\alpha} - \vec{\beta}$ is

(A) 6 sq. unit
(B) 3 sq. unit
(C) 2 sq. unit
(D) 5 sq. unit

37. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then $\left|\frac{\hat{a} - \hat{b}}{2}\right|$ is equal to

(A) $\sin \frac{\theta}{2}$
(B) $\sin \theta$
(C) $2 \sin \theta$
(D) $\sin 2\theta$

38. Let $\vec{\beta} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{\gamma} = \hat{i} + 3\hat{k}$. If $\hat{\alpha}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]$ is

(A) -1

(B) $\sqrt{10} + \sqrt{6}$

(C) $\sqrt{59}$

(D) $\sqrt{60}$

39. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors such that $\vec{a} + \vec{b} + \vec{c} = \alpha\vec{d}$ and $\vec{b} + \vec{c} + \vec{d} = \beta\vec{a}$, then $\vec{a} + \vec{b} + \vec{c} + \vec{d}$ equals to

(A) $\vec{o}$

(B) $\alpha\vec{a}$

(C) $\beta\vec{b}$

(D) $(\alpha + \beta)\vec{c}$

40. Let $\vec{\alpha}, \vec{\beta}$ and $\vec{\gamma}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{\alpha} + 2\vec{\beta}$ is collinear with $\vec{\gamma}$ then $\vec{\alpha} + 2\vec{\beta} + 6\vec{\gamma}$ equals

(A) $\lambda\vec{\alpha}$ ($\lambda \neq 0$, a scalar)

(B) $\lambda\vec{\beta}$ ($\lambda \neq 0$, a scalar)

(C) $\lambda\vec{\gamma}$ ($\lambda \neq 0$, a scalar)

(D) $\vec{o}$

41. Find the point of intersection of the lines

$$\frac{x-a_1}{a_2} = \frac{y-b_1}{b_2} = \frac{z-c_1}{c_2} \text{ and } \frac{x-a_2}{a_1} = \frac{y-b_2}{b_1} = \frac{z-c_2}{c_1}$$

(A) $(a_1 - a_2, b_1 - b_2, c_1 - c_2)$

(B) $(a_1 + a_2, b_1 + b_2, c_1 + c_2)$

(C) $\left(a_1 - \frac{a_2}{2}, b_1 - \frac{b_2}{2}, c_1 - \frac{c_2}{2}\right)$

(D) $\left(a_1 + \frac{a_2}{2}, b_1 + \frac{b_2}{2}, c_1 + \frac{c_2}{2}\right)$

42. The angle between a normal to the plane $2x - y + 2z - 1 = 0$ and the x-axis is

(A) $\cos^{-1}\frac{1}{3}$

(B) $\cos^{-1}\frac{2}{3}$

(C) $\cos^{-1}\frac{3}{4}$

(D) $\cos^{-1}\frac{3}{5}$

43. The equation of the plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ parallel to x-axis is

(A) $y + 3z + 6 = 0$

(B) $y + 3z - 6 = 0$

(C) $y - 3z + 6 = 0$

(D) $y - 3z - 6 = 0$

44. Envelope of the family of lines $x \operatorname{cosec} \theta - y \cot \theta = c$ (θ being the parameter) is

(A) $x^2 + y^2 = 2c^2$ (B) $x^2 - y^2 = c^2$

(C) $xy = c^2$ (D) $x^2 + 2y^2 = c^2$

45. $\int_{-1/2}^{1/2} \cos x \log \frac{1+x}{1-x} dx$ is

(A) 1 (B) 0

(C) 2 (D) $\frac{1}{2}$

46. Area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$ is

(A) $\frac{\pi a^2}{8}$ square units (B) $\frac{3\pi a^2}{8}$ square units

(C) $\frac{\pi a^2}{4}$ square units (D) $\frac{\pi a^2}{3}$ square units

47. $\lim_{n \to \infty} \frac{\sqrt[n]{n!}}{n}$ ($n \in N$) is

(A) -1 (B) e^{-1}

(C) -2 (D) e^{-2}

48. Let $f(x) = \begin{cases} 2^x + 1, & -1 \le x < 0 \\ 2^x, & x = 0 \\ 2^x - 1, & 0 < x \le 1 \end{cases}$

Then,

(A) $f(x)$ is continuous & bounded in $[-1, 1]$

(B) $f(x)$ is bounded in $[-1, 1]$, but never reaches is upper bound & lower bound

(C) $f(x)$ is decreasing throughout $[-1, 1]$

(D) $f(x)$ has points of infinite discontinuity in $[-1, 1]$.

49. Consider the curve $y = x^3$ & two points on it $A(-1, 1)$ & $B(2, 8)$. Then

(A) there is no point on the curve at which the tangent is parallel to chord AB.

(B) there are two points on the curve at which the tangents are parallel to chord AB

(C) there is only one point at which the tangent is parallel to chord AB

(D) tangent cannot be drawn at any point on the curve.

50. Let $p_n(x)$, $n \ge 0$ be a polynomial defined by

$p_n(x) = x\, p_{n-1}(x) - p_{n-2}(x)$ for $n \ge 2$,

$p_0(x) = 1$, $p_1(x) = x$

Then $p_{10}(0)$ equals

(A) 0 (B) 10

(C) 1 (D) −1

51. The equation $x^3y + xy^3 + xy = 0$ represents

(A) a circle (B) a circle & a pair of straight lines

(C) a rectangular hyperbola (D) a pair of straight lines

52. $\int_0^1 \log x \, dx$

(A) does not exist

(B) is a proper integral & its value is –1

(C) is an improper integral & its value is –1

(D) value is 0

53. The number of roots of the equation $x^2 + \sin^2 x = 1$ in the closed & bounded interval $\left[0, \frac{\pi}{2}\right]$ is

(A) 0

(B) 1

(C) 2

(D) 3

54. Let $f(x) = \cos x - 1 + \frac{x^2}{2}$, then

(A) $f(x)$ is increasing on the real line.

(B) $f(x)$ is decreasing on the real line.

(C) $f(x)$ is increasing in $-\infty < x < 0$ and is decreasing in $0 \leq x < \infty$

(D) $f(x)$ is decreasing in $-\infty < x < 0$ and is increasing in $0 \leq x < \infty$

55. Let $f(x, y) = \begin{cases} \frac{x^3 - xy^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0 & , x^2 + y^2 = 0 \end{cases}$

(A) $\lim_{(x,y) \to (0,0)} f(x, y)$ does not exist

(B) f is continuous at (0, 0)

(C) $f_x(0, 0) = f_y(0, 0)$

(D) $xf_x + yf_y = 5f(x, y)$

56. Let t > 0 be fixed. Then $\lim_{n \to \infty} \frac{1}{n}\left\{\sin\frac{t}{n} + \sin\frac{2t}{n} + ... + \sin\frac{(n-1)t}{n}\right\}$ is

(A) $\frac{\sin t}{t^2}$

(B) $\frac{1-\cos t}{t}$

(C) $\frac{\cos t}{t^2}$

(D) $\frac{\sin 2t}{t^2}$

57. Pick out the series which is absolutely convergent :

(A) $\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$

(B) $\sum_{n=1}^{\infty}(-1)^n \frac{(n+1)^n}{n^{n+1}}$

(C) $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+2}$

(D) $\sum_{n=1}^{\infty}(-1)^n \frac{2n+1}{2n+3}$

58. The curve whose subtangent is of constant length p is

(A) $x = Ce^{-p/y}$

(B) $xy = p^2$

(C) $x = py$

(D) $y = Ce^{x/p}$

59. The orthogonal trajectory of the family $y^2 = ax$ is

(A) $2x^2 + y^2 = C^2$

(B) $x^2 - 2y^2 = D$

(C) $x^2 + xy + 2y^2 = C$

(D) $x^2 + y^2 = C^2$

60. $f(x, y) = x^2 + 5y^2 - 6x + 10y + 6$, minimum value of $f(x, y)$ is

(A) –7

(B) –5

(C) –3

(D) –12

61. The equation $1 + xy - ln(e^{xy} + e^{-xy}) = 0$ defines y as an implicit function of x. Then $\frac{dy}{dx}$ is

(A) $\frac{1}{x^2}$ (B) $\frac{y}{x}$

(C) $-\frac{y}{x}$ (D) $-\frac{x}{y}$

62. Let $z = e^{xy}$ where $y = \varphi(x)$, then

(A) $\frac{dz}{dx}$ is irrelevant (B) $\frac{dz}{dx} - \frac{\partial z}{\partial x}$ is constant

(C) $\frac{dz}{dx} \neq \frac{\partial z}{\partial x}$ (D) $\frac{dz}{dx} = \frac{\partial z}{\partial x}$

63. $\lim_{x \to 1}\left(\frac{x}{x-1} - \frac{1}{ln\, x}\right)$

(A) does not exist (B) $\frac{1}{2}$

(C) $\frac{1}{3}$ (D) ∞

64. The function y is defined implicitly by the equation $x^2 + 2xy + y^2 - 4x + 2y - 2 = 0$. Then at (1, 1), $\frac{d^2y}{dx^2}$

(A) does not exist (B) 1

(C) $\frac{1}{3}$ (D) 0

65. The values of b and c for which the parabola $y = x^2 + bx + c$ is tangent to $y = x$ at the point (1, 1) are

(A) $b = -1, c = 1$ (B) $b = -1, c = -1$

(C) $b = 1, c = 0$ (D) $b = 1 = c$

66. Let $y = \cos^{-1}\left\{\frac{3x + 4\sqrt{(1-x^2)}}{5}\right\}$, $|x| < 1$. Then $\frac{dy}{dx}$ is

(A) $2x\sqrt{(1-x^2)}$

(B) $\frac{-1}{\sqrt{(1-x^2)}}$

(C) $\frac{1}{\sqrt{(1-x^2)}}$

(D) $\frac{2x}{\sqrt{(1-x^2)}}$

67. Consider the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

(A) Axes are the asymptotes

(B) $y = x$ is only one asymptote

(C) the curve has no asymptote

(D) $y = -x$ is only one asymptote

68. $\lim_{x \to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$

(A) does not exist

(B) $e^{\frac{2}{\pi}}$

(C) e^2

(D) e^{π}

69. $\lim_{x \to 1}\left(\frac{1+x}{2+x}\right)^{\frac{(1-\sqrt{x})}{(1-x)}}$

(A) does not exist

(B) e^2

(C) $\frac{1}{2}$

(D) $\left(\frac{2}{3}\right)^{\frac{1}{2}}$

70. $t^n x^{n+1}\, dt + t^{n+1} x^n (1 - 3t^2 x^2)\, dx = 0$ is exact. Then

(A) $n = 1$

(B) $n = 3$

(C) $n = -3$

(D) $n = -1$

71. Let $f : [1, e] \to \mathbb{R}$ be defined by $f(x) = x^2 \ln x$, then

(A) $\exists$ two critical points inside the given interval

(B) no critical point inside the indicated interval but least value is attained at left end point and greatest value at right end point

(C) no absolute maximum & minimum value exist

(D) critical points exist in $\left[\frac{e}{2}, \frac{3}{2}\right]$

72. Let f' exist in [0, 1]. Then $f(1) - f(0) = \frac{f'(x)}{2x}$

(A) has no solution

(B) has at least one solution

(C) is an identity

(D) condition given is not enough to conclude anything regarding the existence of solution

73. Angle of interaction between the curves $x^2 + y^2 - 4x = 1$ & $x^2 + y^2 + 2y = 9$ is

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

74. Let $\varphi = (ax^2 + 2hxy + by^2)\, f\left(\frac{y}{x}\right) + \frac{1}{Ax + By} g\left(\frac{y}{x}\right)$ where f and g have continuous second order partial derivatives. Then $x^2\varphi_{xx} + 2xy\varphi_{xy} + y^2\varphi_{yy}$ is

(A) 0

(B) $2\varphi(x, y)$

(C) $\varphi(x, y)$

(D) $4\varphi(x, y)$

75. Let $f(x, y) = (|x + y| + x + y)^k$, $(x, y) \in S \subset \mathbb{R}^2$

(A) f_x and f_y exist at (0, 0) $\forall$ k $\in \mathbb{R}$

(B) f_x and f_y exist at (0, 0) $\forall$ k $\in$ (0, 1)

(C) f_x and f_y exist at (0, 0) $\forall$ k > 1

(D) f_x and f_y exist at (0, 0) only for $k > \frac{1}{2}$

76. Let $I = \int_2^\infty \frac{x^2 dx}{\sqrt{(x^7 + 1)}}$, $J = \int_2^\infty \frac{x^3 dx}{\sqrt{(x^7 + 1)}}$, then

(A) I & J both are convergent

(B) I is convergent but J is not so

(C) I is divergent but J is convergent

(D) Neither I nor J is convergent

77. Let C'(ℝ) denote the set of all continuously differentiable real valued functions defined on the real line. Define $A = \left\{f : C'(\mathbb{R}) : f(0) = 0,\ f(1) = 1,\ |f'(x)| \le \frac{1}{2}\ \forall x\right\}$ then

(A) $A = \phi$

(B) A is non-void finite set

(C) A is infinite set

(D) A is singleton

78. Which of the following statement is true?

(A) It is possible to make the substitution $x = \sec t$ in the integral $I = \int_0^1 \sqrt{(x^2 + 1)}\, dx$

(B) It is not possible to make the substitution $x = \sec t$ in the integral $I = \int_0^1 \sqrt{(x^2 + 1)}\, dx$

(C) It is possible to put $t = 1/x$ in evaluating $\int_{-1}^{1} \frac{dx}{1 + x^4}$

(D) It is possible to write $\int_0^\infty \frac{dx}{x} = \int_0^1 \frac{dx}{x} + \int_1^\infty \frac{dx}{x}$

79. $I = \int_0^3 \frac{x\,dx}{\sqrt{x+1}+\sqrt{5x+1}}$ is equal to

(A) $\frac{7}{8}$

(B) $\frac{3}{8}$

(C) $\frac{9}{13}$

(D) $\frac{14}{15}$

80. Let $y = x^n \{c_1 \cos(ln\, x) + c_2 \sin(ln\, x)\}$ where c_1, c_2 are real constants. Then $x^2y_2 + B(x)y_1 + C(x)y = 0$ holds for

(A) $B = (1-2n)x$, $C = 1 + n^2$

(B) $B = (1 + 2n)$, $C = (2-n)x$

(C) $B = 2n^2$, $C = -3n$

(D) $B = n^3$, $C = 1 + n$

81. Which of the following equation represent bounded area in the xy-plane ?

(A) $xy = 1$

(B) $17x^2 - 12xy + 8y^2 + 46x - 28y + 17 = 0$

(C) $x = 5y^2 - 6y + 1$

(D) $x^2 + xy - 2y^2 + 5x + y - 6 = 0$

82. For $\frac{dy}{dx} = \frac{4y}{x} + x\sqrt{y}$, the solution is

(A) $y = x^4\left(\frac{1}{2} ln x + c\right)^2$

(B) $x = e^y + c$

(C) $x\sqrt{y} = e^y + c$

(D) $y\sqrt{x} + xy = A$

83. In the equation $\frac{d^2y}{dx^2}+\frac{2x}{1+x^2}\frac{dy}{dx}+\frac{y}{(1+x^2)^2}=0$, the independent variable is changed to θ by $x = \tan\theta$. The transformed equation is

(A) $\frac{d^2y}{d\theta^2}-\frac{dy}{d\theta}+2y=0$

(B) $\frac{d^2y}{d\theta^2}+\frac{dy}{d\theta}-2y=0$

(C) $\frac{d^2y}{d\theta^2}+y=0$

(D) $\frac{d^2y}{d\theta^2}-\left(\frac{dy}{d\theta}\right)^2+5y=0$

84. $\lim_{n\to\infty}\left(\sqrt{n^2+n}-\sqrt{n^2+1}\right)$, $(n \in \mathbb{N})$

(A) does not exist

(B) is 0

(C) is $-\frac{1}{2}$ and can be evaluated by L' Hospital's Rule

(D) is $\frac{1}{2}$ and cannot be evaluated by L' Hospital's Rule

85. Area of the triangle formed by the tangent at any point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the axes is maximum for the point

(A) $(a\sqrt{2}, b\sqrt{2})$

(B) $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$

(C) (2a, 2b)

(D) $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

86. Let $A=\begin{pmatrix}1 & 1\\ x & y\\ x^2 & y^2\end{pmatrix}$, $x, y \in \mathbb{R}$. Then AA^T is

(A) Skew symmetric

(B) Orthogonal $\forall x, y \in \mathbb{R}$

(C) Singular

(D) Symmetric

87. The number of complex numbers ω such that $|\omega| = 1$ & imaginary part of ω^4 is 0, is

(A) 4

(B) 2

(C) 8

(D) infinitely many

88. Let $f : X \to Y$ and A, B are two non-void subsets of X. Then

(A) $f(A) \subset f(B)$, if $A \subset B$

(B) $f(A) \supset f(B)$, if $A \subset B$

(C) $f(A) - f(B) = f(A - B)$

(D) $f(A) - f(B) \supset f(A - B)$

89. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x(x + 1)(x - 1)$, then

(A) f is one-one & onto

(B) f is neither one-one nor onto

(C) f is onto but not one-one

(D) f is neither one-one nor onto

90. Let p(x) be continuous function which is positive $\forall x$ and $\int_2^3 p(x)dx = c\int_0^2 p\left(\frac{x+4}{2}\right)dx$ then

(A) $c = 4$

(B) $c = \frac{1}{2}$

(C) $c = \frac{1}{4}$

(D) $c = 2$

91. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$. The matrix equation $AX = B$ holds if

(A) $X = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

(B) $X = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(C) $X = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

(D) $X = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

92. $\begin{vmatrix} a+b & 2a+b & 3a+b \\ 2a+b & 3a+b & 4a+b \\ 4a+b & 5a+b & 6a+b \end{vmatrix}$ is equal to

(A) $a + b$

(B) $2a + 3b$

(C) $3a + 4b$

(D) 0

93. Solution of $y(1 + x)\,dx + x(1 + y)dy = 0$ is

(A) $xy + \log(x + y) = a \quad \forall x > 0, y > 0$

(B) $x - y + \log\left(\frac{x}{y}\right) = c \quad \forall x > 0, y > 0$

(C) $x + y + \log(xy) = c \quad \forall x > 0, y > 0$

(D) $\frac{x}{y} + \log(xy) = c \quad \forall x > 0, y > 0$

94. $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$

(A) does not exist

(B) exists & is 1

(C) $6\sqrt{\pi}\Gamma\left(\frac{2}{3}\right)/\Gamma\left(\frac{1}{6}\right)$

(D) $\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3}\right)}{5}$

95. Consider the equation $3x^5 + 15x - 8 = 0$. The equation

(A) has only one real root

(B) has more than one real root

(C) has 3 real roots

(D) has 2 real roots

96. Let A, B, C be non-void sets and U be universal set. Then $(A' \cap B' \cap C) \cup (B \cap C) \cup (A \cap C)$ is

(A) A

(B) B

(C) C

(D) $A \cap C$

97. Let $u(x, y) = \dfrac{x^2y^2}{x+y}$, $x + y \neq 0$. Then $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$

(A) does not exist

(B) is 6u

(C) is 0

(D) is $\dfrac{u}{3}$

98. The differential equation of which $y^2 = 4a(x + a)$ is a solution, is given by

(A) $y\left(\dfrac{dy}{dx}\right)^2 - 2x\dfrac{dy}{dx} + 3y = 0$

(B) $y\left[1-\left(\dfrac{dy}{dx}\right)^2\right] = 2x\dfrac{dy}{dx}$

(C) $\left(\dfrac{dy}{dx}\right)^2 - 4\dfrac{dy}{dx} + 6y = 0$

(D) $\dfrac{dy}{dx} = \sqrt{(y^2 - 4x^2)}$

99. $\displaystyle\lim_{(x, y)\to(0, 0)} \frac{\sqrt{(x^2y^2+1)}-1}{x^2+y^2}$

(A) does not exist

(B) is 0

(C) is 1

(D) is ∞

100. Let $f(x) = 2x^2 - \ln x$

(A) f increases for $0 < x < \frac{1}{2}$

(B) f decreases for $0 < x < \frac{1}{2}$

(C) f decreases for $-\frac{1}{2} < x < \frac{1}{2}$

(D) f increases for $x < \frac{1}{4}$

SPACE FOR ROUGH WORK